FST 3-2 Notes
Topic: The Graph-Translation Theorem
GOAL:
Show that the substitution of x - h for x and y - k for y in an equation for a relation has the effect of translating a graph h units to the right and k units up.

SPUR Objectives

C Use the Graph-Translation Theorem to find transformation images.

D Describe the effects of translations on functions and their graphs.

J Apply the Graph-Translation Theorem to make or identify graphs.

Vocabulary

transformation
preimage
image
translation

Definition of Translation

A **translation** in the plane is a transformation that maps each point (x, y) onto $(x + h, y + k)$, where h and k are constant.

Instruction

$+h$ right $+k$ up
$-h$ left $-k$ down

1) Under a translation, the image of (0, 0) is (12, 25).

right 12, up 25

   a) Find a rule for this translation.

$$(x, y) \rightarrow (x+12, y+25)$$

   b) Find the image (x', y') of (-2, -8) under this translation.

$$(-2, -8) \rightarrow (-2+12, -8+25)$$
$$\boxed{(10, 17)}$$

On your own:

1) Under a translation, the image of (0, 0) is (-12, 5).

left 12, up 5

   a) Find a rule for this translation.

$$(x, y) \rightarrow (x-12, y+5)$$

   b) Find the image of (6, -10) under this translation.

$$(6, -10) \rightarrow (6-12, -10+5)$$
$$\boxed{(-6, -5)}$$

Instruction

2) Compare the graphs of $y = x^3$ and $y - 25 = (x - 12)^3$.

a) Graph it

$$y = (x-12)^3 + 25$$

b) Describe the translation

right 12, up 25

c) Find the rule for this translation

$$(x, y) \rightarrow (x+12, y+25)$$

WINDOW
X min = -10
X max = 20
Y min = -10
Y max = 30

3) If the graph of $y = |x|$ is translated 2 units up and 3 units to the left, what is an equation for its image?

$$(x, y) \rightarrow (x - 3, y + 2)$$
$$y - 2 = |x + 3| \quad \text{or} \quad y = |x + 3| + 2$$

On your own:

2) Compare the graphs of $y = x^2$, *and* $y = (x + 4.2)^2 - 5$.

left 4.2, down 5

3) If the graph of $y = \sqrt{x}$ is translated 2 units down and 5 units to the right, what is an equation for its image?

$$y = \sqrt{x - 5} - 2$$
$$\text{or}$$
$$y + 2 = \sqrt{x - 5}$$

Instruction

a) If the graph of $y = \sqrt{x}$ is translated 2 units down and 5 units to the right, what is the translation rule?

$$(x, y) = (x + 5, y - 2)$$

Graph-Translation Theorem

Given a preimage graph described by a sentence in x and y, the following two processes yield the same image:
(1) replacing x by $x - h$ and y by $y - k$ in the sentence;
(2) applying the translation $(x, y) \rightarrow (x + h, y + k)$ to the preimage graph.
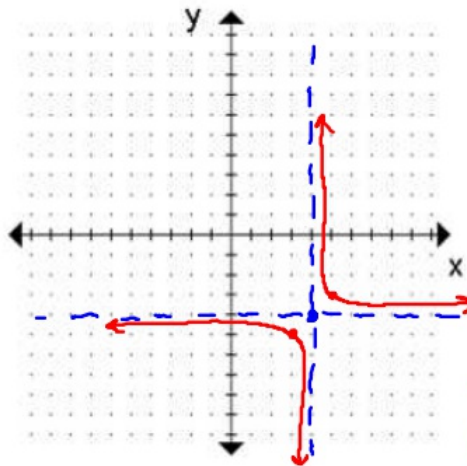
- For $T(x, y) = (x + h, y + k)$ the image of $y = f(x)$ is
  $$y - k = f(x - h)$$
  or $\quad y = f(x - h) + k$

Example: If the graph of $y = -\dfrac{1}{2x^2}$ is translated 8 units up and 17 units to the left, what is an equation for its image?

$$-\frac{1}{2(x+17)^2} + 8$$

Example: Sketch the graph of $y = \dfrac{1}{(x-4)} - 4$

right 4
down 4

Asymptotes
$x = 4$
$y = -4$

D: $\{x \mid x \neq 4\}$
R: $\{y \mid y \neq -4\}$

A good graph meets the following criteria:
- Axes are labeled appropriately, with the scales shown.
- The characteristics of the graph can be seen (appropriate window).

  -asymptotes

  -discontinutities

  -changes in direction
- The intercepts are shown.